

DUSO Mathematics League 2014 - 2015

Contest #3.

Calculators are not permitted on this contest.

Part I.

ALGEBRA I

Time Limit: 10 minutes

The word "compute" calls for an exact answer in simplest form.

3 - 1. Compute the greatest integer value of x that solves $\frac{6}{5}x + 2 > \frac{8}{3}x - 4$.

3 - 2. Compute all ordered triples (x, y, z) of real numbers that solve the following:

$$xy + yz = 6$$

$$xz + xy = -28$$

$$yz + xz = -38$$

Part II.

GEOMETRY

Time Limit: 10 minutes

The word "compute" calls for an exact answer in simplest form.

3 - 3. In right triangle TRI with hypotenuse \overline{TR} , $\sin T + \sin R = 1.3450$. Find $\cos T + \cos R$ to four decimal places.

3 - 4. Triangle BOX has vertices at $B(-2, 2)$, $O(1, 2)$, and $X(1, 6)$. After a glide reflection, the coordinates of the vertices of the image triangle are $B''(6, 2)$, $O''(6, 5)$, and $X''(10, 5)$. Determine the equation of the line used in the glide reflection (with coefficients in simplest form) and the direction of the translation (that is, if the translation is $T_{a,b}$, compute a and b).

Part III.

ALGEBRA II / ADVANCED TOPICS

Time Limit: 10 minutes

The word "compute" calls for an exact answer in simplest form.

3 - 5. The circle with equation $x^2 + y^2 + 6x - 4y - 23 = 0$ has a center at (h, k) and a radius of r . Compute $h + k + r$.

3 - 6. Compute all values of x that satisfy the following equation: $|x| + 5 - |x + 5| = 10$

R-1. The diagonals of square $ABCD$ intersect at E . How many distinct triangles can be formed whose vertices are at A , B , C , D , or E ?

R-2. Let N be the number you will receive. The number $2^N \cdot 5^4$ is written as a decimal number. Compute the sum of the digits of this number.

R-3. Let N be the number you will receive. The point P is one-third of the way from (N, N) to $(16, -5)$. Compute the coordinates of P .

R-4. Let P be the point you will receive. The graph of the parabola with equation $y = ax^2 + bx + c$ passes through $(6, 3)$, $(8, 4)$, and P . Compute the product abc .

R-5. Let N be the number you will receive. A jar contains 15 balls, of which N are black and the rest are red. If three balls are chosen from the jar without replacement, compute the probability that all three are red.